A New Beam-Scanning Technique by Controlling the Coupling Angle in a Coupled Oscillator Array

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***Abstract*—This letter presents a new technique for electronic beam scanning of a coupled oscillator array. A constant phase progression in a coupled oscillator array is achieved by controlling the coupling phase of the outermost coupling circuits only, while that of the innermost coupling circuits is zero and all free-running frequencies of the oscillators are the same. Analytical solution of nonlinear phase dynamic equations changes periodically as a piecewise-linear function of coupling phase only. The theory developed in this letter is verified using a four-element oscillator array operating at 6.16 GHz. The full scan range is measured to be -17° to 18° off broadside. This scan range is very close to the theoretically achievable scan range of -19.2° to 19.2° for this array.**

***Index Terms*—Constant phase progression, coupled oscillator array, electronic beam scanning.**

I. Introduction

QUASI-OPTICAL coupled oscillator array has been used for millimeter-wave power combinings [1], [2], mode locking for pulse generations [3], and electronic beam scannings [4]–[6], etc. In a previously proposed technique by Stephan [4], two signals with a controlled phase difference $\Delta\psi$ are injected into the opposite ends of the array. The resulting phase progression between each of the N oscillators is then found to be $\Delta\psi/(N+1)$. In a proposed technique by Liao [5], [6] and York [7], as shown in Fig. 1(a), a constant phase progression which is independent of the number of oscillators is established by controlling the free-running frequency of the end oscillators only. However, much of phase progression variation occurs for frequency shifts near the edge of locking range, where the phase noise of the array output will be increased. Moreover, the amplitude of the end oscillators changes significantly over this regime. Consequently, the coupled oscillator array cannot lock effectively to a common frequency in this regime. These problems cause the full scan range to be much smaller than a theoretically achievable one. In this letter, a constant phase progression which can solve above-mentioned problems is established by controlling the coupling angle but not free-running frequency. The theory developed in this letter is verified by comparing with experimental data using a four-element oscillator array operating at 6.16 GHz.

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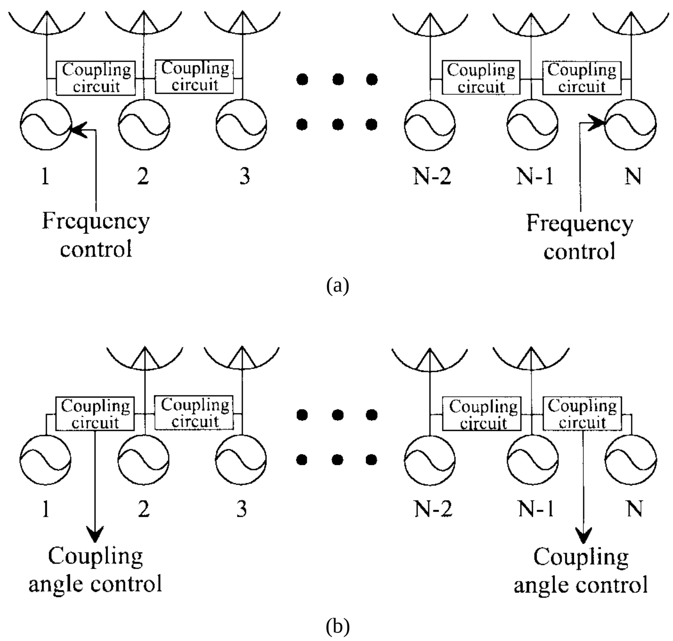


Fig. 1. (a) Block diagram of a previously phased array system for beam scanning and (b) illustration of the proposed technique.

II. Theoretical Development

The phase dynamics of N coupled oscillators with nearest neighbor coupling can be modeled by using Van der Pol equation [5], [6] as

$$\frac{d\theta_i}{dt} = \omega_i - \frac{\omega_i}{2Q} \sum_{j=i-1,\, j\neq i}^{i+1} \epsilon_{ij} \frac{A_j}{A_i} \sin(\Phi_{ij} + \theta_i - \theta_j), \quad i = 1, 2, \cdots, N \tag{1}$$

where ω_i, θ_i, A_i, and Q are free-running frequency, instantaneous phase, instantaneous amplitude, and quality factor of the ith oscillator, while ϵ_{ij} and Φ_{ij} are the magnitude and phase of the coupling circuit between oscillators i and j, respectively.

Reciprocity will hold so that $\epsilon_{ij} = \epsilon_{ji}$ and $\Phi_{ij} = \Phi_{ji}$. For identical oscillators, $\omega_1 = \omega_2 = \cdots = \omega_N = \omega$, $\epsilon_{12} = \epsilon_{23} = \cdots = \epsilon_{(N-1)N} = \epsilon$. Many coupling structures are conceivable. Following the coupling method described in [7], each oscillator is coupled to its two nearest neighbor via a resistively loaded microstrip line. If all innermost coupling circuits have one wavelength long microstrip line at free-running frequency, one can set the coupling phase of all innermost coupling circuits to zero: $\Phi_{23} = \Phi_{34} = \cdots = \Phi_{(N-1)N} = 0$. Following the

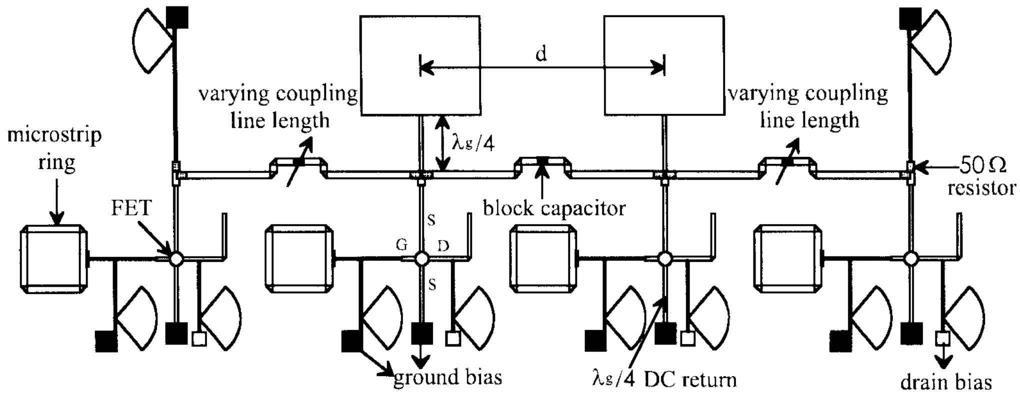


Fig. 2. Diagram illustrating the experimental four-element oscillator array.

procedure, one assumes this to be the case. Furthermore, it is also assumed that the mutual interaction between oscillators i and j is coupled weakly, where $\epsilon_{ij} \ll 1$. Then, instantaneous amplitudes will remain close to their free-running amplitudes: $A_1 \approx A_2 \approx \cdots \approx A_N$. By defining $\theta_i - \theta_{i+1} = \Delta\theta_i$, (1) becomes

$$
\begin{aligned}
\frac{d\Delta\theta_1}{dt} &= -2\Delta\omega_m \cos\Phi_{12} \sin\Delta\theta_1 + \Delta\omega_m \sin\Delta\theta_2 \\
\frac{d\Delta\theta_2}{dt} &= -\Delta\omega_m \sin(\Phi_{12} - \Delta\theta_1) - 2\Delta\omega_m \sin\Delta\theta_2 \\
&\quad + \Delta\omega_m \sin\Delta\theta_3 \\
\frac{d\Delta\theta_i}{dt} &= -\Delta\omega_m \sin\Delta\theta_{i-1} - 2\Delta\omega_m \sin\Delta\theta_i \\
&\quad + \Delta\omega_m \sin\Delta\theta_{i+1}, \qquad \text{for } 3 \le i \le N-3 \\
\frac{d\Delta\theta_{N-2}}{dt} &= +\Delta\omega_m \sin[\Phi_{(N-1)N} + \Delta\theta_{N-1}] \\
&\quad - 2\Delta\omega_m \sin\Delta\theta_{N-2} + \Delta\omega_m \sin\Delta\theta_{N-3} \\
\frac{d\Delta\theta_{N-1}}{dt} &= -2\Delta\omega_m \cos\Phi_{(N-1)N} \sin\Delta\theta_{N-1} \\
&\quad + \Delta\omega_m \sin\Delta\theta_{N-2}
\end{aligned} \tag{2}
$$

where $\Delta\omega_m = \epsilon\omega/2Q$ is locking range. If all of the oscillators are mutually synchronized, the derivatives of $\Delta\theta_i$ will then be equal to zero. Therefore, one can obtain $N-1$ nonlinear equations represented by

$$
\begin{aligned}
\sin\Delta\theta_1 &= \frac{\sin\Delta\theta_2}{2\cos\Phi_{12}} \\
\sin\Delta\theta_2 &= \frac{\sin\Delta\theta_3 - \sin(\Delta\theta_1 - \Phi_{12})}{2} \\
\sin\Delta\theta_i &= \frac{\sin\Delta\theta_{i-1} + \sin\Delta\theta_{i+1}}{2}, \\
&\quad \text{for } 3 \le i \le (N-3) \\
\sin\Delta\theta_{N-2} &= \frac{\sin\Delta\theta_{N-3} + \sin[\Delta\theta_{N-1} + \Phi_{(N-1)N}]}{2} \\
\sin\Delta\theta_{N-1} &= \frac{\sin\Delta\theta_{N-2}}{2\cos\Phi_{(N-1)N}}.
\end{aligned} \tag{3}
$$

The steady-state solution for (3) can be obtained with ordinary nonlinear root-finding algorithm. For the special case of $\Phi_{12} = -\Phi_{(N-1)N} = \Phi$, analytical solution for (3) can be found intuitively by inspecting (3) carefully without using numerical method as

$$
\begin{aligned}
\Delta\theta_1 &= \Delta\theta_{N-1} = -\Phi, \qquad \text{for } |\Phi| \le \pi \\
\Delta\theta &= \Delta\theta_2 = \Delta\theta_3 = \cdots = \Delta\theta_{N-2} \\
&= -2|\Phi|\text{sng}(\Phi), \qquad \text{for } |\Phi| \le \pi/4 \\
\Delta\theta &= \Delta\theta_2 = \Delta\theta_3 = \cdots = \Delta\theta_{N-2} \\
&= 2(|\Phi| - \pi/2)\text{sng}(\Phi), \qquad \text{for } \pi/4 \le |\Phi| \le 3\pi/4 \\
\Delta\theta &= \Delta\theta_2 = \Delta\theta_3 = \cdots = \Delta\theta_{N-2} \\
&= 2(\pi - |\Phi|)\text{sng}(\Phi), \qquad \text{for } 3\pi/4 \le |\Phi| \le \pi
\end{aligned} \tag{4}
$$

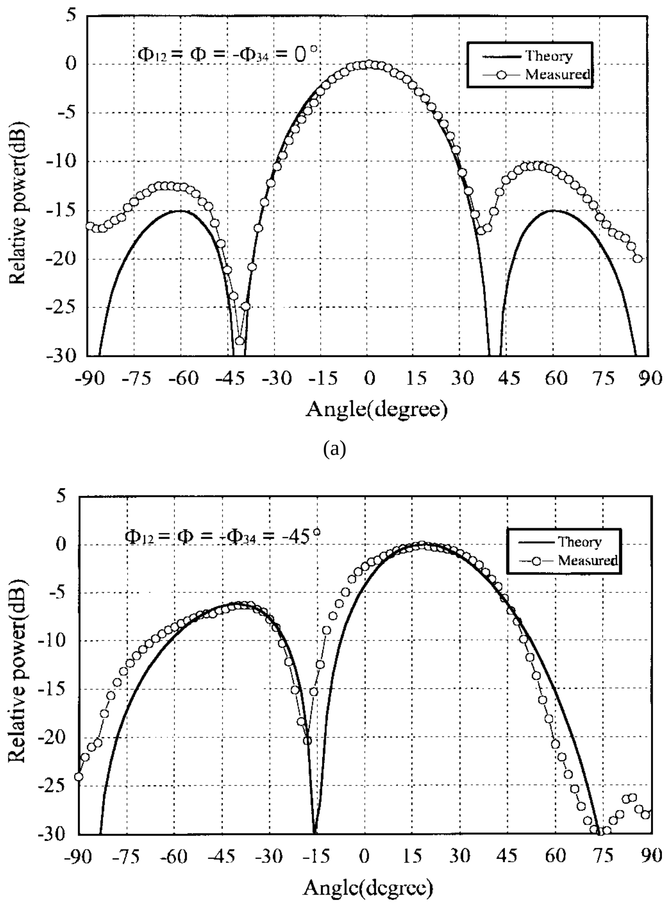
where

$$
\text{sgn}(\Phi) = \begin{cases} 1, & \text{if } \Phi \ge 0 \\ -1, & \text{if } \Phi < 0. \end{cases}
$$

Equation (4) shows that the analytical solution changes periodically as a piecewise-linear function of coupling phase only. It is found that a constant phase progression $\Delta\theta$ from 2 to $(N-1)$th oscillator can be achieved by controlling the coupling phase Φ of the outmost coupling circuits only, as shown Fig. 1(b) by equal amounts but opposite signs, but not the free-running frequencies of end elements only, as shown in Fig. 1(a). Note that $\Delta\theta$ is independent of the number of oscillators array and locking range. This solution is in full agreement with numerical solution obtained by fourth-order Runga–Kutta routine.

III. Experiment and Results

The theoretical predictions are tested using a four-element oscillators array as shown in Fig. 2. The oscillators use NE76038d packaged MESFET's as active device and microstrip ring as resonator, respectively. As output load impedance of outermost oscillators, a 50-Ω chip resistor is used. Using a $\lambda_g/4$ high-impedance line, the output load impedance is equalized for the two innermost oscillators attached to patch antenna with that of end element oscillators at free-running frequency. The width and length of each patch antenna are 18 and 14.6 mm, respectively. The circuit is fabricated on 0.787-mm-thick Taconic TLX-9 board of $\epsilon_r = 2.5$. The oscillators in the array are coupled together by 100-Ω-microstrip line, resistively loaded with two 50-Ω chip resistor. All free-running frequencies in the array are set to the



(a) (b)

Fig. 3. Comparison of theoretical and experimental results for two different scan angles by controlling outermost coupling angle by equal amounts but opposite signs while all free-running frequencies are same and coupling phase of innermost coupling circuits is zero. (a) Measured and theoretical patterns for $\Phi = 0°$ and (b) measured and theoretical patterns for $\Phi = -45°$.

same 6.16 GHz by adjusting the drain bias of each element while every other element is unpowered. The coupling phase of the outermost coupling circuits is manipulated by varying the length of the line while that of the innermost coupling circuit is set to zero. The one wavelength-long innermost coupling line is arranged with an element separation of 37.2 mm, or 0.76 λ_0 at 6.16 GHz. When the oscillators are all powered on and mutually locked, the measured output frequency is 6.1595 GHz.

Several array patterns are measured for various coupling angle distributions, and some of the results are shown in Figs. 3 and 4. The full scan range is measured to be $-17°$ to $18°$ off broadside. This scan range is smaller than the theoretically achievable scan range of $-19.2°$ to $19.2°$ for this array, mainly because the coupling phase of the outermost coupling circuit is not exactly controlled by varying the length of the coupling line in the oscillators array. As mentioned earlier, however, much of phase progression variation occurs for frequency shifts near the edge of locking range in previously proposed techniques [5]–[7]. The amplitude of the end oscillators also changes significantly over this regime. Consequently, the coupled oscillator array cannot lock effectively to a common

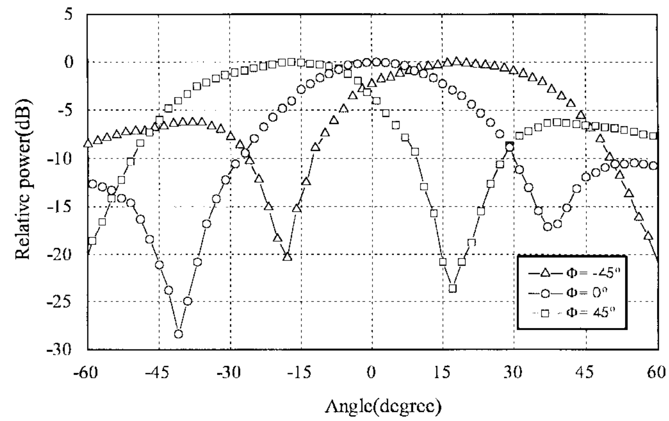


Fig. 4. Comparison of measured radiation patterns at three different scan angles. Continuous beam is possible from $-17°$ to $18°$ by adjusting the outermost coupling angle by equal amounts but opposite signs, which is very close to the maximum $\pm 19.2°$ predicted by the theory.

frequency in this regime. These problems cause the full scan range to be much smaller than a theoretically achievable one. On the other hand, this problem does not occur in the proposed technique because all free-running frequencies of the oscillators are set to same values. Therefore, the accomplished scan range is very closer to the theoretically achievable scan range than the previously proposed techniques [5]–[7].

IV. Conclusion

A new technique for electronic beam scanning is presented. A constant phase progression which is independent of the number of oscillators and locking range is accomplished by controlling the coupling phase of the outmost coupling circuits only, while that of the innermost coupling circuits is zero and all free-running frequencies of the oscillators are the same. This technique is tested using an experiment of a four-element oscillator array. The radiation pattern of patch antenna attached to innermost two-element oscillators could be continuously steered over a range of angles from $-17°$ to $18°$ off broadside. This scan range is very close to the theoretically achievable scan range.

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